

On the use of the average large separation in surface layer independent model fitting (Research Note)

Ian W. Roxburgh

Astronomy Unit, Queen Mary University of London, Mile End Road, London E1 4NS, UK. e-mail: I.W.Roxburgh@qmul.ac.uk

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ABSTRACT

The physics of the outer layers of a star are not well understood but these layers make a major contribution to the large separation. We quantify this using stellar models and show that the contribution ranges from 6% from the outer 0.1% of the radius to 30% from the outer 5%. and therefore argue that the large separation should not be used as a constraint on surface layer independent model fitting. The mass and luminosity are independent of the outer layers and can be used as constraints, the mass being determined from binarity or from surface gravity and radius. The radius can be used as a constraint but with enhanced error estimates. We briefly consider the determination of the large separation for α Cen A and find that mass derived from surface gravity is closer to the binary mass than that derived from the large separation.

Key words. stars: oscillations, - asteroseismology - stars: interiors - methods: analytical - methods: numerical

1. Introduction

It has long been appreciated that modelling the outer layers of a star is subject to many uncertainties due to our poor understanding of the physical processes in these layers (cf Christensen-Dalsgaard et al, 1988, Dziembowski et al 1988). These include modelling convection, convective overshooting, non-adiabatic effects on both convection and oscillations, turbulent pressure, the equation of state, diffusion, mild turbulence, magnetic fields, rotation and global circulation. All these factors impact on the oscillation frequencies of a model star, and therefore hinder efforts to find stellar models whose frequencies best fit an observed frequency set.

One way of seeking to overcome these problems is the “frequency offset technique” (Kjeldsen et al 2008), in which the difference between the observed solar frequencies and those of a “best solar model” is scaled by a single factor (determined by the average frequency and large separation) and applied to other stars when seeking a best fit model. This assumes that the many differences in the properties of the outer layers of stars can all be captured in a single scaling factor; such an assumption remains to be verified.

An alternative approach is to use techniques which are (almost) independent of the structure of the outer layers: these fit combinations and/or properties of the frequencies that only depend on the structure of the inner layers, and consequently can only give information on the interior of a star and not on the outer layers. Such techniques include the ratio of small to large separations (Roxburgh and Vorontsov 2003a, 2013) and matching of phases in the outer layers (Roxburgh and Vorontsov 2003b, Roxburgh 2010)

When seeking to derive a best fit model for a given set of observed frequencies it is usual to impose some global constraints on the models such as luminosity, effective temperature, surface gravity, radius, and composition, and in particular the average

value of the large separation $\Delta = \langle \nu_{n+1,\ell} - \nu_{n,\ell} \rangle$ of an observed frequency set $\{\nu_{n,\ell}\}$, often estimating the mass of the star from the (approximate) “scaling relation” $\Delta \propto (M/R^3)^{1/2}$ so that

$$\frac{M}{M_\odot} = \left(\frac{\Delta}{\Delta_\odot}\right)^2 \left(\frac{R}{R_\odot}\right)^3 \quad (1)$$

where $\Delta_\odot (\approx 135 \mu\text{Hz})$ is the solar value.

As we show below, the outer layers of a star make a major contribution to the value of the large separation; since the objective of surface-layer independent model fitting is to subtract out the effect of the outer layers of the star it would be inconsistent to constrain the model fitting by requiring that the model had the observed large separation.

2. Simple analysis

In the first order asymptotic approximation the oscillation frequencies satisfy the equation (Vandakurov 1967, Tassoul 1980)

$$\nu_{n,\ell} = \Delta_T(n + \ell/2 + \epsilon) \quad \text{where} \quad \Delta_T = \frac{1}{2T} \quad \text{and} \quad T = \int_0^R \frac{dr}{c(r)} \quad (2)$$

is the acoustic radius of the star, c the sound speed ($c^2 = \Gamma_1 P/\rho$) and ϵ is a constant. Since c is smallest in the outer layers the contribution of these layers to T and hence Δ is significant, so uncertainties in the structure of the outer layers can produce significant uncertainty in Δ_T .

To quantify this we separate T into the contribution t_f from the layers below a fractional radius $x_f = r_f/R$, and $\tau_f = T - t_f$ from the layers above x_f . From the definition of Δ_T (Eqn 2) it follows that

$$\Delta_T = 2(t_f + \tau_f)\Delta_T^2 \quad (3)$$

so the contribution to Δ from the layers above x_f is $\delta\Delta = 2\tau_f\Delta_T^2$. In Table 1 we show the contribution $\delta\Delta$ (in μHz) from the layers

above x_f for a model main sequence star of $1.10M_\odot$ with central hydrogen abundance $X_c = 0.250$ and $\Delta_T = 119.6\mu\text{Hz}$

Table 1. Contribution $\delta\Delta$ to Δ_T versus x_f ($M = 1.10M_\odot$)

x_f	0.0	0.50	0.90	0.95	0.97	0.99	0.995	0.999
$\delta\Delta$	119.6	91.16	45.4	33.5	26.4	15.7	10.7	4.4

As can be seen from Table 1, just the outer 0.1% of the star contributes 4% to Δ_T and the outer 5% contributes 30%, so errors in modelling the outer layers can produce a substantial change in Δ_T , much greater than the error estimates on the mean large separation of typical frequency sets obtained by the CoRoT or Kepler missions (and of α Cen A&B and β Hydri), which are typically $0.1 - 0.4\mu\text{Hz}$ (cf Creevey et al 2013).

3. Full analysis

The first order asymptotic relation (Eqn 2) is, in general, a poor approximation and the mean large separation Δ around typical observed n values ($n \sim 20$) differs from the value given by the acoustic radius Δ_T . For the $1.10M_\odot$ stellar model $\Delta \approx 117.0\mu\text{Hz}$, whereas $\Delta_T = 119.6\mu\text{Hz}$. We therefore use a full non-asymptotic analysis to further study the effect of uncertainties in the outer layers, replacing the asymptotic relation (2) by the expression

$$\nu_{n,\ell} = \Delta(n + \ell/2 + \epsilon_{n\ell}) \quad (4)$$

where the *phase shifts* $\epsilon_{n\ell}(\nu_{n,\ell})$ are defined by this relation once an average value of Δ has been specified.

As shown by Roxburgh and Vorontsov (2000, 2003a), Roxburgh (2009a), on matching the solution of the oscillation equations integrated away from the centre with the solution integrated in from the surface at any intermediate acoustic radius t , the eigenfrequencies of a star satisfy the equation

$$2\pi T\nu = \pi[n + \ell/2] + \alpha_\ell(\nu, t) - \delta_\ell(\nu, t) \quad \text{where} \quad T = \int_0^{R_s} \frac{dr}{c} \quad (5)$$

is the total acoustic radius of the star (from the centre $r = 0$ to the top of the atmosphere $r = R_s$). This equation is identical to Eqn 4, with $\Delta = 1/(2T)$ and $\epsilon_{n\ell} = (\alpha_\ell - \delta_\ell)/\pi$. A different choice of Δ just adds a term linear in ν to the $\epsilon_{n\ell}$.

Here $\delta_\ell(\nu, t)$, $\alpha_\ell(\nu, t)$ are inner and outer *phase shifts* defined by the equations

$$\frac{2\pi\nu\psi}{d\psi/dt} = \tan[2\pi\nu t - \ell\pi/2 + \delta_\ell(\nu, t)] \quad t \leq t_f \quad (6a)$$

$$\frac{2\pi\nu\psi}{d\psi/dt} = -\tan[2\pi\nu\tau - \alpha_\ell(\nu, t)] \quad t \geq t_f \quad (6b)$$

where $\psi = rp'/(\rho c)^{1/2}$ with $p'(r)$ an Eulerian pressure perturbation, $t = \int_0^r dr/c$ the acoustic radius at r , $\tau = T - t$ the acoustic depth, and t_f any arbitrarily chosen acoustic radius. For modes of degree $\ell = 0, 1$, where the 4th order system of oscillation equation collapse to second order, the $\alpha_\ell(\nu)$ at any acoustic radius t_f are determined solely by the structure of the layers above t_f , and $\delta_\ell(\nu)$ at any t_f are determined solely by the structure interior to t_f . This is also a very good approximation for modes of degree $\ell = 2, 3$ provided t_f is taken in the outer layers where the density is small.

To demonstrate this we take the model of a main sequence star (ModelA) of mass $1.10M_\odot$, initial composition $X = 0.72$, $Z = 0.02$ evolved to a central hydrogen abundance $X_c =$

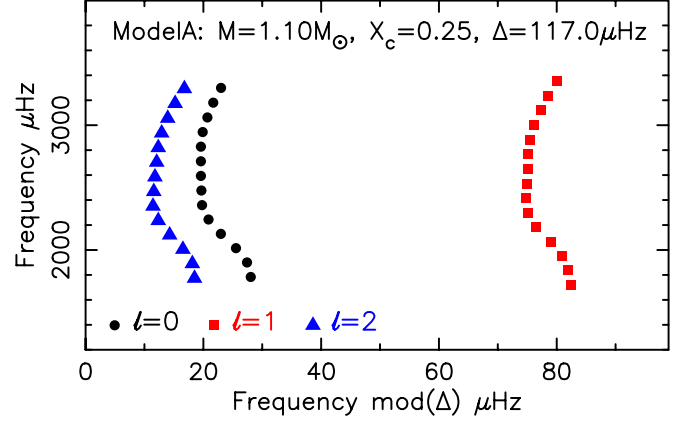


Fig. 1. Echelle diagram of the frequencies of modelA, a main sequence star of mass $1.10M_\odot$ evolved to a central hydrogen abundance $X_c = 0.25$.

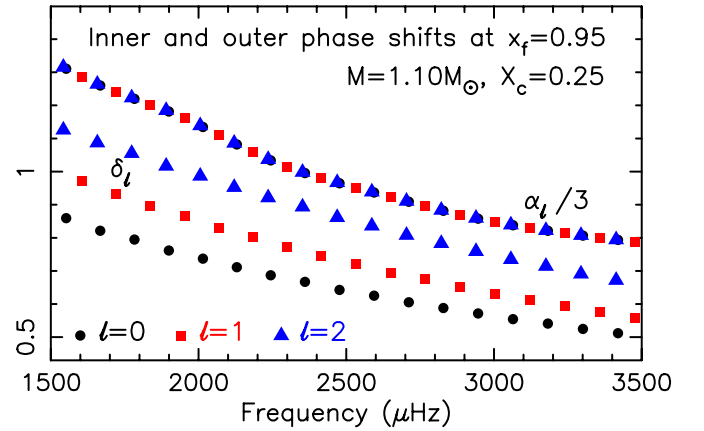


Fig. 2. Inner phase shifts $\delta_\ell(\nu)$ and outer phase shifts $\alpha_\ell(\nu)$ for a stellar model of $1.10M_\odot$ and modes of degree $\ell = 0, 1, 2$. Note that the $\alpha_\ell(\nu)$ all lie on the same curve.

0.25, whose frequencies for $\ell = 0, 1, 2$ are shown Fig 1 in a traditional echelle diagram. Fig 2 shows the $\alpha_\ell(\nu)$ and $\delta_\ell(\nu)$ for this model at a fitting radius t_f , corresponding to a fractional radius $x_f = r_f/R = 0.95$. The fact that to high accuracy all the $\alpha_\ell(\nu)$ lie on the same curve independent of ℓ is the basis of surface layer independent model fitting techniques.

The phases $\alpha_\ell(\nu, t)$, $\delta_\ell(\nu, t)$ can be evaluated for any acoustic radius t_f and any frequency ν , but for an eigenfrequency ψ and $d\psi/dt$ must be continuous. Since $\tan A + \tan B = 0$ gives $\tan(A + B) = 0$, it follows that $A + B = n\pi$, hence, at any t_f , an eigenfrequency satisfies the condition (as in Eqn 5)

$$2\pi\nu_{n\ell}(t_f + \tau_f) - \ell\pi/2 + \delta_\ell(\nu_{n\ell}, t_f) - \alpha_\ell(\nu_{n\ell}, t_f) = n\pi \quad (7)$$

Subtracting this equation from the corresponding equation for $n + 1$ we obtain

$$\frac{1}{\Delta_{n\ell}} = \left[\left(2t_f + \frac{1}{\pi} \frac{\partial \delta_\ell}{\partial \nu} \right)_i + \left(2\tau_f - \frac{1}{\pi} \frac{\partial \alpha_\ell}{\partial \nu} \right)_o \right] \quad (8)$$

where

$$\frac{\partial \delta_\ell}{\partial \nu} = \frac{\delta_\ell(\nu_{n+1,\ell}) - \delta_\ell(\nu_{n,\ell})}{\nu_{n+1,\ell} - \nu_{n,\ell}} \quad \frac{\partial \alpha_\ell}{\partial \nu} = \frac{\alpha_\ell(\nu_{n+1,\ell}) - \alpha_\ell(\nu_{n,\ell})}{\nu_{n+1,\ell} - \nu_{n,\ell}} \quad (9)$$

The first term in brackets with subscript i in Eqn 8 is determined solely by the structure interior to the fitting point t_f and the second term with subscript o is determined solely by the structure

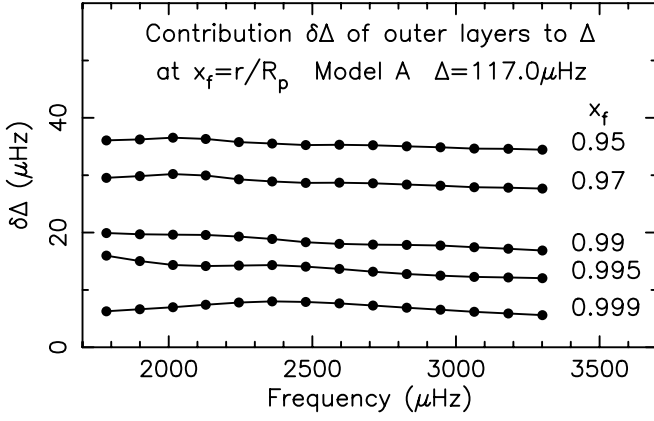


Fig. 3. Contribution $\delta\Delta$ of the outer layers to the large separations $\Delta_{n,0}$ for different depths, for a stellar model of $1.10M_{\odot}$ with $\Delta \sim 117\mu\text{Hz}$. R_p is the photospheric radius.

exterior to t_f . On multiplying Eqn 8 by $\Delta_{n\ell}^2$ we deduce that the contribution of the outer layers to $\Delta_{n\ell}$ is

$$\delta\Delta_{n\ell}(t_f) = \left(2\tau_f - \frac{1}{\pi} \frac{\partial\alpha_{\ell}}{\partial\nu}\right) \Delta_{n\ell}^2 \quad (10)$$

Fig 3 shows $\delta\Delta$ for modes $\ell = 0$ and $n = 14, 27$, and for different depths, in terms of the fractional radii $x_f = r/R_p = 0.95, 0.97, 0.99, 0.995, 0.999$. Here the outer 0.1% of the radius contributes 7% to Δ and the outer 5% contributes 30%, in broad agreement with the results of the simple analysis in section 2. So inaccuracies in the modelling of the outer layers can have a large effect on the value of Δ .

These results apply equally to more evolved stars. To show this we undertook the same analysis for model B, a highly evolved post main sequence star of mass $1.15M_{\odot}$ with initial composition $X = 0.72$, $Z = 0.015$ in the shell burning phase moving over to the red giant branch. The model has many mixed modes as can be seen in the echelle diagram (Fig 4). The phase shifts are shown in Fig 5: the inner phase shifts $\delta_{n\ell}$ for $\ell = 1, 2$ no longer lie on smooth curves but the outer phase shifts $\alpha_{n\ell}$ still all lie on a single curve for all ℓ . In Fig 6 we give the contribution of the outer layers to the large separation Δ for 14 $\ell = 0$ modes with $n = 9, 22$, as a function of fractional radius x_f . Here the outer 0.1% of the radius contributes 6.5% to Δ and the outer 5% some 32%, more or less the same as for Model A.

4. Global constraints on surface layer independent model fitting

The above analysis demonstrates that the even small differences in the structure of the outer layers of a star can make a significant difference to the value of the large separation Δ . When using surface layer independent model fitting (separation ratios or phase matching), one is seeking to subtract the effect of the outer layers so it is inconsistent to constrain the search by requiring the model fit the observed large separation.

One can ask what global constraints should one impose. The two obvious global constraints are the mass M and luminosity L , since these are essentially determined solely by the inner structure of the star. If the star has a measured parallax then the luminosity can be estimated, but the mass is only known for stars in binary systems, the prime example being α Cen A&B.

In principle one can estimate the mass from spectroscopically determined surface gravity g , and surface radius R estimated either from interferometry or from L, T_{eff} . Such a mass

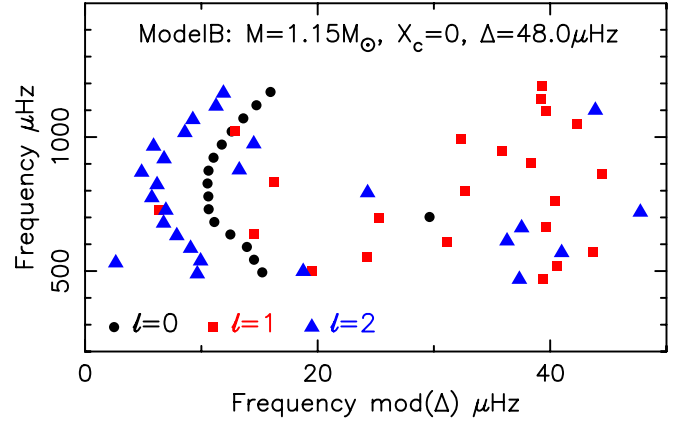


Fig. 4. Echelle diagram of the frequencies of modeB, a $1.15M_{\odot}$ post main sequence model in the shell burning phase - the model has many mixed modes for both $\ell = 1$ and $\ell = 2$.

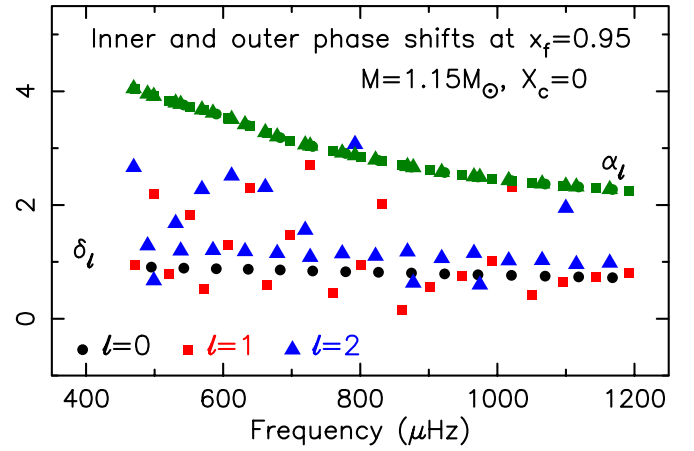


Fig. 5. Inner phase shifts $\delta_{\ell}(\nu)$ and outer phase shifts $\alpha_{\ell}(\nu)$ for a stellar model of $1.15M_{\odot}$ and modes of degree $\ell = 0, 1, 2$. Note that the $\alpha_{\ell}(\nu)$ (in green) still all lie on a single curve.

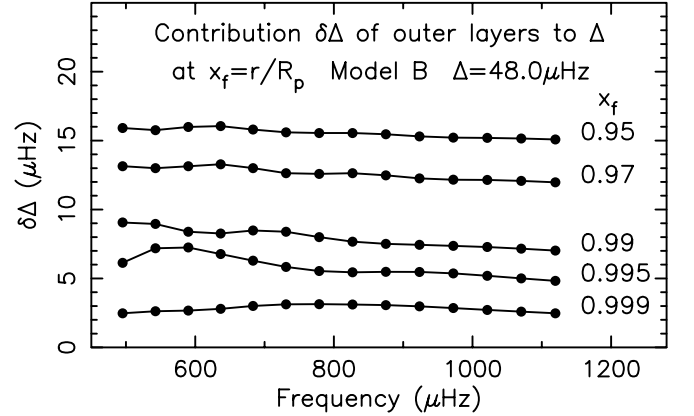


Fig. 6. Contribution $\delta\Delta$ of the outer layers to the large separations $\Delta_{n,0}$ for different depths, for a modelB of $1.15M_{\odot}$ with $\Delta \sim 48\mu\text{Hz}$. R_p is the photospheric radius.

estimate is independent of uncertainties due to surface layer contributions to R , but often g is not known to high enough precision to provide a useful constraint. The outer layer contributions to R are much smaller than those to Δ ; the layers above $x_f = 0.99$ contribute 1% to the radius but 17% to the large separation, so one could reasonably impose a radius constraint but with an enhanced error estimate to allow for the unknown contribution of the outer layers.

5. α Cen A

The fundamental properties of α Cen A have been determined to high precision; these and the derived radius are listed in Table 2: the binary mass is from Pourbaix et al. (2002); parallax from Soderhjelm (1999); angular diameter from Kervella et al (2003); $\log g$ from Bruntt et al (2010).

Table 2. α Cen A&B: observational input

	$M/M_{\odot}(B)$	π (mas)	θ (mas)	$\log g$ (cm/s ²)	R/R_{\odot}
A	1.105 ± 0.007	747.1 ± 1.2	8.511 ± 0.020	4.309 ± 0.055	1.224 ± 0.003

There have been several investigations to determine the frequencies of α Cen A (Bouchy and Carrier 2002, Bedding et al 2004, Fletcher et al, 2006, Bazott al 2007), and more recently by de Meulenaer et al (2010) who combined the time series from Bouchy and Carrier with those from Bedding. The different investigations are not in total agreement with each other indicating either the difficulty in extracting frequencies and estimating uncertainties, or possibly a variation in time, or both. The average large separation Δ estimated from these frequencies are all in the vicinity of $106\mu\text{Hz}$ but vary by $\sim 0.5\mu\text{Hz}$.

We here estimate the average large separation directly from the widowed autocorrelation of the de Meulenaer combined time series as this does not depend on uncertainties in frequency determination, and is more robust since it combines all the data into the determination of the one quantity - the average Δ (Roxburgh and Vorontsov 2006, Roxburgh 2009b). To be specific we used a \sin^2 window of FWHM $= 8\Delta$ centred on the peak of maximum power, here found to be $\nu_{\max} = 2384\mu\text{Hz}$. The resulting $\Delta = 106.1\mu\text{Hz}$. The value varies with the choice of window width and ν_{\max} by $\sim 0.2\mu\text{Hz}$ which we take as an error estimate on Δ . We did the same analysis using the SPM solar time series (kindly supplied by T Appourchaux) to determine the solar value of $\Delta_{\odot} = 134.97 \pm 0.1\mu\text{Hz}$.

Table 3. α Cen A&B: derived radii and masses

	$M/M_{\odot}(B)$	$\Delta(\mu\text{Hz})$	$M(g)/M_{\odot}$	$M(\Delta)/M_{\odot}$
A	1.105 ± 0.007	106.1 ± 0.2	1.115 ± 0.151	1.134 ± 0.011

The resulting mass $M(\Delta)$ obtained from the large separation Δ using the scaling relation (Eqn 1) is shown in Table 3 together with the mass $M(g)$ derived from $\log g$. (We took the following solar values: $M_{\odot}=1.98855 \cdot 10^{33}\text{gm}$, $R_{\odot}=6.9599 \cdot 10^{10}\text{cm}$.)

It is interesting to note that mass $M(g)$ is more compatible with the dynamically determined value than is $M(\Delta)$, or equivalently that the spectroscopic $\log g$ is closer to the dynamical value than is the value obtained from the large separation scaling relation (Eqn 1).

Not too much weight should be given to this result as the combined time series is still only 11.4 days in duration so the accuracy of our value of Δ is open to question. To address this issue we determined the values of the average solar large separation Δ_{\odot} from 50 non overlapping 11.4 day time series taken from the SPM data. Here we found that Δ_{\odot} varied between 134.66 and $135.31\mu\text{Hz}$ indicating an uncertainty of $\pm 0.4\mu\text{Hz}$ whereas the same analysis on 20 non overlapping 150 day time strings found a variation between 134.87 and 135.06, consistent with the previously estimated uncertainty of $0.1\mu\text{Hz}$. This suggests that the uncertainty on the 11.4 day estimate of Δ for α Cen A should be enhanced by a factor ~ 4 to $0.8\mu\text{Hz}$ which would give an estimate of $M(\Delta)/M_{\odot} = 1.134 \pm 0.020$ almost within overlapping 1σ errors of the binary mass of $M_A/M_{\odot} = 1.105 \pm 0.007$.

This also suggests that the values of the frequencies and their error estimates for α Cen A could be different for different short duration time series, which may in part explain the difference between the estimated frequencies obtained by different authors.

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